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"Clashing Symmetries" In Unified Descriptions of Electromagnetic and
Weak Interactions, and the Case for the Han-Nambu Model[†]

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ABSTRACT

Difficulties encountered in constructing unified descriptions of electromagnetic and weak interactions are simply characterized in terms of the "clash" between the new symmetries needed and the old symmetries already present in hadron dynamics. Unobserved new particles and new decay modes are predicted and must be explained away. The Han-Nambu model is suggested as one which already contains some of the features which are added ad hoc in other models.

Many problems arising in the attempts to unify descriptions of electromagnetic and weak interactions¹ are illuminated by examination from the point of view of internal symmetries. These problems can be characterized as clashes between the new symmetries needed in the unified description and the old established symmetries of hadron physics. The incompatibility between the old and new symmetries is resolved by postulating a higher symmetry which includes both, but not as a simple direct product. But higher symmetries introduce new symmetry generators which imply new degrees of freedom. Also new particles not yet observed are required to fill larger "supermultiplets". The new particles should be degenerate with observed particles in some symmetry limit, but their masses must be pushed up very high by symmetry breaking effects to explain the failure to observe them.

As there is no unique choice indicated for the proper higher symmetry to resolve the symmetry clash, all proposed models have some degree of arbitrariness. Unfortunately, these higher symmetries do not give new relations with previously unrelated experimental facts, like the relation between the decays of ^{14}O and the muon given by the CVC theory. Thus such relations are not available to test and compare different models. Instead, the new symmetries all predict the existence of particles and sometimes also of transition processes which have not yet been observed. These "bad" predictions are not useful for experi-

mental tests of the models. They only provide exercises for theorists to find ingenious mechanisms and cancellations to explain away the predictions of unobserved phenomena.

The purpose of this note is to point out some examples of these symmetry clashes and to suggest serious consideration of the Han-Nambu model² which already has a suitable higher symmetry invented for the purposes of hadron dynamics. Although the incorporation of this model into the unified description of weak and electromagnetic interactions is just as ugly, ad hoc and arbitrary as any other model, all degrees of freedom and extra particle states have already been put in by the hadron physicists, and no additional ones are needed for the electromagnetic and weak properties. How important this is is a matter of taste.

The first symmetry clash is with parity. The only observed neutral current is the electromagnetic current which conserves parity; i. e., it is coupled equally to left-handed and right-handed particles. The only observed charged currents are the parity-violating weak currents which are coupled only to left-handed particles. These observed currents can all be put into the same symmetry scheme only by adding either (1) a neutral parity-violating current or (2) a charged right-handed current. In either case some excuse must be found to explain the failure to observe the additional current at the present level of experiment. At present it seems that experimental evidence is gradually

pushing the neutral weak current out of consideration. The right-handed charged currents are still admissible, because they can be postulated to always involve heavy leptons which have so far not yet been observed. This suggests that parity violation is a "low-energy" phenomenon which disappears at high energies where the mass differences between the electron, muon and heavy leptons are negligible. Thus parity violation is associated with the breaking of a higher symmetry in which the leptons are degenerate and classified in the same multiplet of the symmetry.

An $O(3)$ or $SU(2)$ symmetry has been suggested³ to give a description with heavy leptons and no new neutral currents. One of the $O(3)$ generators is the electric charge, and the vector bosons are classified in an $O(3)$ triplet. Leptons and hadrons are classified in various $O(3)$ representations. This insures that the neutral vector boson is coupled to the electric charge and that the only neutral current in the theory is the electromagnetic current.

This $O(3)$ symmetry clashes with $SU(3)$ symmetry and leads to difficulties in the inclusion of hadrons in the $O(3)$ scheme. The clash is between incompatible requirements for the electric charge. In the conventional $SU(3)$ classification of hadrons, isospin is an $SU(2)$ subgroup, but I_z is not quite the electric charge. The electric charge differs from I_z by a function of the hypercharge. The electric charge is a generator of $SU(3)$ with integral and third-integral eigenvalues, rather than

integral and half integral. This operator cannot be an $O(3)$ generator. Thus there is no $O(3)$ or $SU(2)$ subgroup of $SU(3)$ in which one generator is exactly equal to the electric charge, as required by the $O(3)$ scheme. Furthermore, quarks with third-integral electric charge cannot be classified in this $O(3)$.

The symmetry clash is resolved by embedding both $SU(3)$ and $O(3)$ into a larger group. In this description the electric charge must be a generator of $O(3)$ and cannot be just a generator of $SU(3)$. It must be a linear combination of isospin, hypercharge and a generator of the higher group. Furthermore, the electric charge must be defined to give integral charges for all fundamental $SU(3)$ triplets in the theory.

This resolution of the $O(3)$ - $SU(3)$ symmetry clash leads to the "charm" problem. The additional generator needed to define electric charge must have a vanishing eigenvalue for all the low-lying observed hadron states whose charge, baryon number and strangeness satisfy the Gell-Mann-Nishijima formula. This has several interesting consequences.

1. New particles must exist which require an additional quantum number, commonly called charm, and a new formula to relate their conserved quantum numbers. The new formula differs from the Gell-Mann-Nishijima formula but reduces to it for the case of uncharmed particles. Some excuse must then be found to push up the masses of these charmed particles to explain why they have not been seen.

2. Charmed pieces of the electromagnetic and weak currents exist which must have vanishing matrix elements between pairs of uncharmed hadron states. These do not contribute to first order weak electromagnetic processes. In higher order calculations, charmed intermediate states can occur, and they can be important in loops, where their high mass may be unimportant. The presence of such states and of the charmed pieces of the currents can provide key factors for canceling divergences, or for introducing additional statistical factors in the calculation of triangle diagrams.

3. Universality of the weak interactions is not simply defined in models containing unobserved charmed states at very high masses. The weak and electromagnetic currents have non-vanishing matrix elements connecting the low-lying hadron states like the nucleon with these unobserved higher states. When a commutation relation between two currents is evaluated by inserting a complete set of intermediate states within the commutator, charmed intermediate states may or may not be included. Conventional universality relations, which do not consider the existence of charmed states, can be interpreted in these new models only by restricting the intermediate states to the subspace of uncharmed states. This is true both in the Adler-Weisberger sum rule and in the CVC relation between the muon and ^{14}O decays. If matrix elements of currents between the nucleon and charmed states are included in these sum rules, they spoil the agreement with experiment. Thus the currents do not satisfy the

Gell-Mann current algebra; it is only the matrix elements in the truncated subspace of uncharmed states which satisfy the algebra.

There are two approaches to the charm problem. One is to add to the conventional $SU(3)$ triplet some new charmed quarks which are singlets in the conventional $SU(3)$. These charmed quarks are assumed to have a higher mass than the conventional $SU(3)$ triplet, so that bound states including charmed quarks have a higher mass and should not yet have been observed. This approach has led to $SU(4)$ and $SU(5)$ models. Another approach is that in the Han-Nambu three-triplet model, in which the additional quarks are also triplets in the conventional $SU(3)$. The three triplets of the Han-Nambu model appear on an equal footing in the low-lying uncharmed hadron states. The difference between charmed and uncharmed states does not appear as a difference between bound states containing charmed or uncharmed quarks, as in the $SU(4)$ -type models. Rather charm is defined by the permutation symmetry in the new degree of freedom describing the three triplets. This is most simply described by an $SU(3)$ group which transforms one ordinary $SU(3)$ triplet to another. The observed uncharmed states are required to be singlets in this new $SU(3)$. The charmed states are those having a different permutation symmetry in the new degree of freedom; they are classified in non-singlet representations of the new $SU(3)$.

The Han-Nambu model has the advantage of already being well

known in the description of the hadron spectrum and having properties already defined by the requirements of hadron physics; namely, by the requirement of integral electric charges and fermi statistics for the three constituents of a baryon. It also keeps all the desirable features of the quark model description of hadron structure, some of which are lost in other models. In particular it keeps the relation between the baryon octet and baryon decuplet in a description where both are made of the same three fundamental objects and differ only by couplings of spin and unitary spin.

The Han-Nambu model can be incorporated into the $O(3)$ scheme without adding any new particles, and without changing the electromagnetic and weak couplings of the fundamental objects in the subspace of uncharmed states. The desired properties under $O(3)$ are obtained by playing with the couplings involving charmed states; i. e. , by adjusting the pieces of the currents which are not singlets under the new $SU(3)$ group. This adjustment procedure has many arbitrary and ad hoc features, but such features seem to be common in all proposed models.

The nine fundamental particles in the Han-Nambu description are five neutral particles and four charged particles, two with charge +1 and two with charge -1. These charges require the $O(3)$ classification of the nine particles to have two vectors to accommodate the four charged particles and three scalars to accommodate the remaining

three neutrals. However, there is no a priori or unique prescription for deciding which pair of charged states with opposite charge belong in the same $O(3)$ vector and which of the five neutral states belongs with them. There are therefore many possible choices for the classification of the nine particles into two $O(3)$ vectors and three scalars. The electromagnetic and the weak currents are linear combinations of singlets and octets in the new $SU(3)$ group in which uncharmed states are singlets. The singlet parts of the currents are responsible for all transitions between observed hadron states. The octet parts connect the low-lying hadrons with charmed states. These are not observed in common transitions, however, the octet part has finite matrix elements between single quark states. This octet part in the electromagnetic current is a singlet in the conventional $SU(3)$ and provides the difference between the integral charged quarks of the Han-Nambu model and the fractionally charged quarks of the Gell-Mann-Zweig model. However, current commutators satisfy the Gell-Mann current algebra calculated with the component which is a singlet in the second $SU(3)$. The full current including the octet part does not satisfy the current algebra.

When the $O(3)$ group is introduced and the currents are defined in the Han-Nambu model to have the proper transformation properties under $O(3)$, they have both singlet and octet components in the second $SU(3)$. Agreement with observed universality requires that the observed singlet part have the proper normalization. This can be achieved only with a

certain amount of arbitrariness.

One example of an explicit representation for the $O(3)$ classification of the Han-Nambu three triplets can be constructed as follows.

We denote the nine fundamental objects as p_i, n_i, λ_i where $i = 1, 2, 3$.

Their electric charges are given by:

$$Q_{p_1} = Q_{p_2} = +1, \quad (1a)$$

$$Q_{n_1} = Q_{n_2} = Q_{\lambda_1} = Q_{\lambda_2} = Q_{p_3} = 0 \quad (1b)$$

$$Q_{n_3} = Q_{\lambda_3} = -1. \quad (1c)$$

We now define two $O(3)$ vectors, as follows:

$$[V_+^1; V_0^1; V_-^1] = [p_1; (1/\sqrt{2})(n_1 + p_3); n_3] \quad (2a)$$

$$[V_+^2; V_0^2; V_-^2] = [p_2; (1/\sqrt{2})(n_2 + \lambda_1); \lambda_3] \quad (2b)$$

The electro-magnetic and weak charges are the generators of the $O(3)$ group. Consider for example the current which lowers the charge by one unit. The eight terms obtained from the vectors (2a) and (2b) can be divided into a singlet and octet parts in the second $SU(3)$. The singlet part, $J^{(1)}$ does not connect charmed and uncharmed states. The octet part, $J^{(8)}$, only connects the nucleon with charmed states and is not observed in present data. These parts of the current can be written schematically as follows: We consider the component which lowers electric charge:

$$J_-^1 = (1/\sqrt{2})(\bar{p}_1 n_1 + \bar{p}_3 n_3 + \bar{p}_2 n_2) \quad (3a)$$

$$J_-^8 = (1/\sqrt{2})(\bar{p}_1 p_3 + \bar{n}_1 n_3 + \bar{p}_2 \lambda_1 + \bar{n}_2 \lambda_3 + \bar{\lambda}_1 \lambda_3). \quad (3b)$$

The normalization factor $\frac{1}{\sqrt{2}}$ in the expression (3a) reflects the character of the neutral member of the two vectors. In every transition only one-half of the neutral member corresponds to an observed singlet transition; one-half goes into the unobserved octet part. Unless this loss of transition strength into the unobserved octet states is somehow compensated in the lepton classification there will be a departure from universality. The particular representation chosen in equations (2) allow universality to be restored by having an unobserved neutral heavy lepton which combines with neutrino in the $O(3)$ classification with equal weight and therefore introducing a similar factor of $\frac{1}{\sqrt{2}}$ in the description of the observed transitions to neutrino final states. However, this does seem rather arbitrary.

There is no essential reason to chose the particular representations given in equations (2) with a "mixing angle" having the factor $\frac{1}{\sqrt{2}}$. If we begin by choosing the positively charged components for the vectors p_1 and p_2 we are free to chose any combination of the five neutral states (1b) to go with p_1 and any other orthogonal combination to go with p_2 . The same is true for the negatively charged states (1c). For each choice there is a singlet and an octet part to the current

analogous to (3a) and (3b) but the normalization factors are different and there are different modifications required in the lepton classification to maintain universality for observed hadron transitions.

Another possibility is

$$[V_+^1; V_0^1; V_-^1] = [p_1; n_1; n_3] \quad (4a)$$

$$[V_+^2; V_0^2; V_-^2] = [p_2; (\lambda_2 \cos \alpha - p_3 \sin \alpha); \lambda_3] \quad (4b)$$

where the missing angle α can be chosen to give the Cabibbo angle for the singlet part of the current as shown below. The Cabibbo angle was neglected in Eqs. (2) and (3). It can be included simply by defining n and λ as the quark states rotated by the Cabibbo angle.

The currents for the choice (4) have the form

$$J_-^1 = (1/3)[\bar{p}_1 n_1 + \bar{p}_2 n_2 + \bar{p}_3 n_3 + (\cos \alpha - \sin \alpha)(\bar{p}_1 \lambda_1 + \bar{p}_2 \lambda_2 + \bar{p}_3 \lambda_3)], \quad (5a)$$

$$J_-^8 = (1/3)[2\bar{p}_1 n_1 - \bar{p}_2 n_2 - \bar{p}_3 n_3 + \cos \alpha (2\bar{p}_2 \lambda_2 - \bar{p}_1 \lambda_1 - \bar{p}_3 \lambda_3) + \sin \alpha (\bar{p}_1 \lambda_1 + \bar{p}_2 \lambda_2 - 2\bar{p}_3 \lambda_3)] + \bar{n}_1 n_3 + \cos \alpha \bar{\lambda}_2 \lambda_3 - \sin \alpha \bar{p}_2 p_3. \quad (5b)$$

The expression (5a) reduces to the Cabibbo current if

$$\cos \alpha - \sin \alpha = \tan \theta_c \quad (6)$$

where θ_c is the Cabibbo angle.

The particular choice given by Eqs. (4) and (5) has the following

interesting property: the n-quarks appear only in V^1 ; the λ -quarks only in V^2 . Since the currents only transform components of V^1 into other components of V^1 and similarly for V^2 , but do not transform components of V^1 into V^2 or vice versa, thus transformations between n-quark and λ -quark states are forbidden to all orders although transformations between either and p-quark states are allowed. This is important to avoid undesirable second-order strangeness-changing transitions.⁴ A detailed analysis of this case is given by Georgi and Glashow.⁵

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